АСТРОФИЗИКА

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SOME COSMOLOGICAL MODELS WITH NEGATIVE POTENTIALS

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We investigate some cosmological models where the effective potential $V(\phi)$ may become negative for some values of field ϕ. Cosmological evolution in models with a minimum at $V(\phi)<0$ is similar in some respects to the evolution in models with potentials unbounded from below. In this case instead of reaching an AdS regime dominated by the negative vacuum energy, the universe may reach a turning point where it's energy density vanishes, and then it contract to a singularity. In some cases such models may lead to a bounce.

Key words: *Cosmological model: Negative Potential: Bounce*

1. *Introduction*. Since the invention of inflationary cosmology, the theory of evolution of scalar fields in an expanding universe has been investigated quite extensively, both at classical level and quantum level [1-3]. While many features of the scalar field cosmology are well understood, the over-all picture remains somewhat incomplete. Felder et al. [4] have extended the investigation of scalar field cosmology to models with negative potentials of the form $V(\phi)=V_0+m^2\phi^2/2$ (Chaotic Inflation). In this paper, we will study the problem of cosmological models with negative potentials for other forms of the potential $V(\phi)$.

There are several reasons to study cosmology with negative potentials. The first reason is related to the cosmological constant problem. In inflationary cosmology, we can choose [1,5]: $V(\phi)=V_0+m^2\phi^2/2$, V_0 is a small cosmological constant. With $V_0>0$, it leads to an expanding universe leading to de Sitter-like state. Why V_0 be so small and positive? What will happen when $V_0<0$? After a long stage of inflation the universe with $V_0<0$ does approach an AdS regime; instead of that it collapses [6]. In [4] the author have studied cosmological behaviour in a large class of the theories with negative potentials and explained why the universe in these theories stops expanding and eventually collapses (Refer also [7]).

Another reason to study theories with negative potentials is provided by cosmology in gauged super-gravity. It has been found that in all known versions of these theories, potentials with extrema of $V(\phi)>0$ are unbounded from below. Despite this fact, such models can, under certain conditions, describe the present stage of acceleration of the universe [6,8].

One more reason is related to a formal connection with warp factor/bulk scalar dynamics in Brane Cosmology. It has been shown that the equations for the warp factor and scalar field in brane cosmology with a scalar field potential $V(\phi)$ are similar to the equations for the scale factor and scalar field in $4D$ cosmology with negative potentials - $V(\phi)$ [9]. Thus there is an interesting relation between negative potentials and warped geometry with positive potentials.

Finally, cosmology with negative potentials $V(\phi)$ is the basis of cyclic universe model [10] based in part on the Ekpyrotic scenario [11]. However, the authors of [10] assumed that the scalar field $V(\phi)$ at large ϕ is positive and nearly constant. As a result, the universe experiences a super-luminal expansion (inflation) that helps to solve some of the cosmological problems.

The idea that the big-bang is not the beginning of the universe but a point of a phase transition is quite interesting [12-16]. Since the idea of the cyclic scenario does require repeated periods of the inflation anyway, it will be nice to avoid the vulnerability of this scenario with respect to the unknown physics at the singularity.

In this paper, we undertake a study of scalar field cosmology with negative potentials for certain forms of $V(\phi)$ used in inflationary cosmology [1-3]. We can describe several regimes that are possible in scalar field cosmology: the universe can be dominated by potential energy, by kinetic energy, by energy density of an oscillating scalar field, or by matter and radiation. We investigate the models for certain choices of $V(\phi)$ and discuss their evolution. In some models with $V(\phi)<0$, there is expansion i.e. $H=\dot{a}/a>0$ and then $H<0$ (contraction), so there is a turn around. In some cases, there may be possibility of a bounce.

The above conclusion can be altered if an account of quantum effects, including particle production near singularity, is taken into consideration.

2. *Cosmological Models*. We take the FRW metric

$$ds^2=dt^2-a(t)^2\left[\frac{dr^2}{1-kr^2}+r^2\left(d\theta^2+\sin^2\theta\, d\phi^2\right)\right] \tag{1}$$

where $k=+1$, 0, -1 and $a(t)$ is the scale factor. For the perfect fluid, energy-momentum tensor is given by

$$T_{ij}=(\rho+p)u_iu_j-pg_{ij}. \tag{2}$$

We take the equation of state as

$$p=\gamma\rho. \tag{3}$$

The Friedmann equation is

$$H^2+\frac{k}{a^2}=\frac{\rho}{3}. \tag{4}$$

Also

$$\dot{H}-\frac{k}{a^2}=-\frac{1}{2}(\rho+p) \tag{5}$$

$$\frac{\ddot{a}}{a}=-\frac{1}{6}(\rho+3p). \tag{6}$$

The Eqs. (4), (5) and (6) can be written as

$$H^2+\frac{k}{a^2}=\frac{1}{6}\dot{\phi}^2+\frac{1}{3}\left(V(\phi)+\rho_\gamma\right) \tag{7}$$

$$\dot{H}-\frac{k}{a^2}=-\frac{1}{2}\left(\dot{\phi}^2+\rho_\gamma(1+\gamma)\right) \tag{8}$$

$$\frac{\ddot{a}}{a}=\frac{1}{3}\left(V(\phi)-\dot{\phi}^2\right)-\frac{1}{6}\left(\rho_\gamma(1+3\gamma)\right). \tag{9}$$

Here, we have assumed $8\pi G=c=1$ in proper unit. The evolution of scalar field ϕ is given by equation

$$\ddot{\phi}+3H\dot{\phi}+\frac{dV(\phi)}{d\phi}=0 \tag{10}$$

where $\dot{\phi}=d\phi/dt$.

We will study four regimes respectively: (i) Universe is dominated by $V(\phi)$, (ii) Universe is dominated by kinetic energy density $\dot{\phi}^2/2$, (iii) The regime when $V(\phi)\approx\dot{\phi}^2/2$, (iv) Universe is dominated by matter/ radiation ρ_γ.

Now we consider these regimes for different choices of potential $V(\phi)$ [1-3].

2.1. *Section A.* We consider

$$V(\phi)=V_0-\frac{1}{2}m^2\phi^2+\frac{\lambda}{4}\phi^4 \tag{11}$$

where $\lambda\leq 1$. We have $V'(\phi)=0$ at $\phi=0$, $\phi=\pm m/\sqrt{\lambda}$. At $\phi=0$, potential function has point of maxima and at $\phi=\pm m/\sqrt{\lambda}$, there is point of minima.

2.1.1. *The energy density is dominated by* $V(\phi)$. In this regime, we have $\dot{\phi}^2/2<<V(\phi)$, $\rho_\gamma<<V(\phi)$ and $|\ddot{\phi}|<<|3H\dot{\phi}|$. Thus, by $p=\dot{\phi}^2/2-V(\phi)$ and $\rho=\dot{\phi}^2/2+V(\phi)$, we have

$$\rho+p=0. \tag{12}$$

For flat FRW case and under the condition $|\phi^4|>>|2m^2\phi^2|>|V_0|$, from eqs. (7) and (10), we have

$$H^2=\frac{\lambda}{12}\phi^4 \tag{13}$$

$$3H\dot{\phi}-m^2\phi+\lambda\phi^3=0. \tag{14}$$

Using Eqs. (13) and (14), we have

$$\phi(t) = \pm\sqrt{\frac{m^2}{\lambda} - \sqrt{\frac{3}{4\lambda}}\exp\left(4\sqrt{\frac{\lambda}{3}}(t_0 - t)\right)} \tag{15}$$

where t_0 is an integration constant. Thus, for large t, $\phi(t) \to \pm m/\sqrt{\lambda}$.

From Eqs. (13) and (15), we obtain the relation

$$a(t) = \exp\left(\frac{m^2}{8\lambda} + \frac{m^2}{\sqrt{12\lambda}} - \frac{\phi^2(t)}{8}\right). \tag{16}$$

For large t, $a(t) \to \exp\left(m^2/\sqrt{12\lambda}\right)$.

The behaviour of scalar field ϕ with cosmic time t for the case 2.1.1, where the energy density is dominated by $V(\phi)$ can be seen in Fig.1, 2. Here, we consider some physically significant values of $\lambda \le 1$, i.e. $\lambda = 0.95$, 0.75, and 0.5. From Fig.1, it is observed that for all given choices of λ, the scalar field ϕ is an increasing function of cosmic time t, if we consider the positive sign in the RHS of equation (15). The rapidity of growth of scalar field is observed at

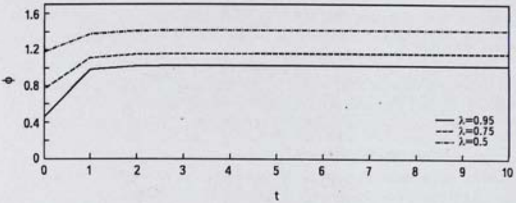

Fig.1. The scalar field ϕ with cosmic time t for $m = 1$, $t_0 = 0$ in positive part of Eq. (15).

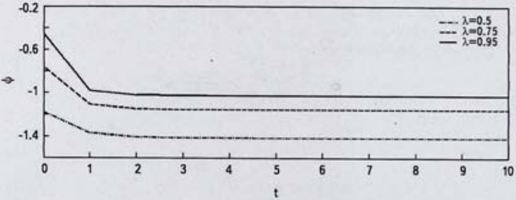

Fig.2. The same as in Fig.1 for negative part of Eq. (15).

an early stage. Later on, this tends asymptotically to a constant for large time t. From Fig.2, it is observed that the scalar field ϕ is decreasing function of cosmic time t, if we consider the negative sign in the RHS of equation (15). The rapidity of decay of scalar field is observed at an early stage. Later on, this tends asymptotically to a constant for large time t.

The behaviour of scalar factor $a(t)$ with cosmic time t for the case 2.1.1, where the energy density is dominated by $V(\phi)$ can be seen in Fig.3. From Fig.3, it is noticed that for all given choices of λ, the scalar factor a is decreasing function of cosmic time t. The rapidity of decay of scalar factor is observed at an early stage. Later on, this tends asymptotically to a constant for large time t, for all above choices of λ.

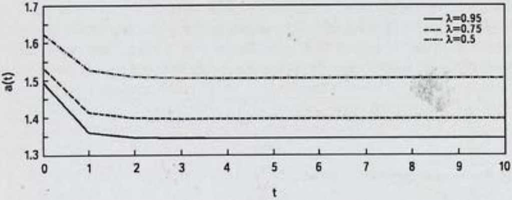

Fig.3. The scale factor $a(t)$ with cosmic time t for $m=1$, $t_0=0$ in Eq. (16).

2.1.2. *The energy density is dominated by* $\dot{\phi}^2/2$. In this regime, we have $V(\phi) << \dot{\phi}^2/2$, $\rho_\gamma << \dot{\phi}^2/2$ and $|\ddot{\phi}| >> dV(\phi)/d\phi$, $|3H\dot{\phi}| >> dV(\phi)/d\phi$. Thus, by $p=\dot{\phi}^2/2-V(\phi)$ and $\rho=\dot{\phi}^2/2+V(\phi)$, we have

$$\rho=p. \tag{17}$$

From Eqs. (7) and (10), we have

$$H=\frac{\dot{\phi}}{\sqrt{6}} \tag{18}$$

$$\ddot{\phi}+3H\dot{\phi}=0. \tag{19}$$

From Eqs. (18) and (19), we have

$$a(t)=\sqrt{\frac{3}{2}}\,a_0 t^{1/3} \tag{20}$$

$$\dot{\phi}=\frac{a_0^3}{a^3}. \tag{21}$$

Also, in other form,

$$a(t) = a_0 \exp\left(\phi_0 + \frac{\phi(t)}{\sqrt{6}}\right) \tag{22}$$

$$\phi = \phi_0 + \sqrt{\frac{2}{3}}\log\left(\frac{t_1}{t}\right) \tag{23}$$

where a_0, t_1 are the constants of integration.

The behaviour of scalar factor $a(t)$ with cosmic time t for the case 2.1.2, where the energy density is dominated by $\dot{\phi}^2/2$ can be seen in Fig.4. From Fig.4, it is observed that the scalar factor a is an increasing function of cosmic time t. The rapidity of growth of scalar factor is observed at an early stage. Later on, this tends asymptotically to a constant for large time t.

The behaviour of scalar field $\phi(t)$ with cosmic time t for the case 2.1.2, where the energy density is dominated by $\dot{\phi}^2/2$ can be seen in Fig.5. Here, we consider some physically significant values of t_1, i.e. $t_1 = 1$, 0.5, and 0.25. From Fig.5,

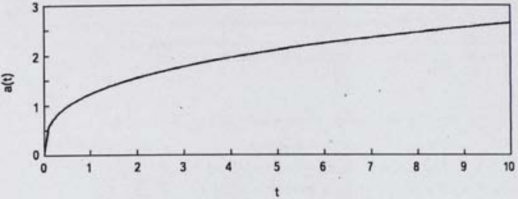

Fig.4. The scale factor $a(t)$ with cosmic time t for $a_0 = 1$ in Eq. (20).

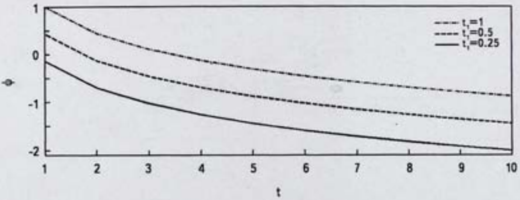

Fig.5. The scalar field ϕ with cosmic time t for $\phi_0 = 1$ in positive part of Eq. (23).

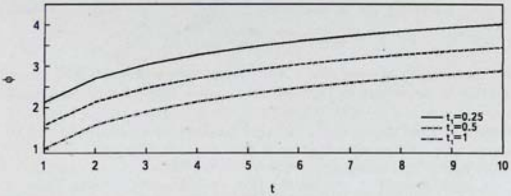

Fig.6. The same as in Fig.5 for negative part of Eq. (23).

it is observed that for all given choices of t_1, the scalar field ϕ is a decreasing function of cosmic time t, if we consider the positive sign in the RHS of Eq. (23). From above Fig.5, it is also noticed that, for large time t, the scalar field ϕ becomes a constant. From Fig.6, it is observed that the scalar field ϕ is an increasing function of cosmic time t, if we consider the negative sign in the RHS of Eq. (23). From above Fig.6, it is also noticed that, for large time t, the scalar field ϕ becomes a constant.

2.1.3. *The regime when* $V(\phi)\approx\dot{\phi}^2/2$. In this regime, we neglect $3H\dot{\phi}$, then from Eq. (10), we have

$$\ddot{\phi}+\frac{dV(\phi)}{d\phi}=0. \tag{24}$$

From Eqs. (11) and (24), we have

$$\log\left[\frac{\sqrt{m^2-\lambda\dot{\phi}^2/2}-m}{\phi}\right]=mt. \tag{25}$$

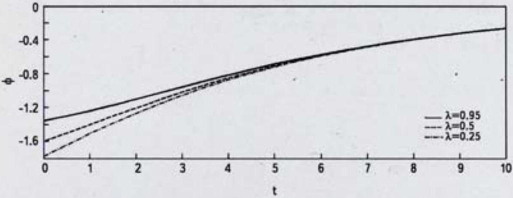

Fig.7. The scalar field ϕ with cosmic time t for $m=1$ in Eq. (26).

In other form, $\phi(t)$ can be written as

$$\phi(t) = -\frac{4m\exp(mt)}{\lambda + 2\exp(2mt)}. \tag{26}$$

The behaviour of scalar field ϕ with cosmic time t for the case 2.1.3, where $V(\phi) \approx \dot{\phi}^2/2$ can be seen in Fig.7. Here, we consider some physically significant values of $\lambda \leq 1$, i.e. $\lambda = 0.95$, 0.5, and 0.25 and $m = 1$. From Fig.7, it is observed that for all given choices of λ, the scalar field ϕ is an increasing function of cosmic time t. From above Fig.7, it is also noticed that, for large time t, the scalar field ϕ is independent on λ.

2.1.4. *Evolution of universe by energy density of matter/radiation* ρ_γ. In this regime, energy density of the universe is dominated by matter with equation of state $p_\gamma = \gamma\rho_\gamma$. The cosmological evolution is in the form, [1]

$$\rho_\gamma = \rho_0 \left[\frac{a(t)}{a_0}\right]^{-3(1+\gamma)} \tag{27}$$

$$a(t) = a_0 \left[\frac{t}{t_0}\right]^{2/3(1+\gamma)} \tag{28}$$

$$H(t) = \frac{2}{3(1+\gamma)t} \tag{29}$$

$$\dot{\phi} = \dot{\phi}_0 \frac{a_0^3}{a^3}. \tag{30}$$

2.2. *Section B*. We consider

$$V(\phi) = M\left(1 - \frac{\alpha}{\phi^n}\right) \tag{31}$$

where $n > 1$ and M is a positive constant.

2.2.1. *The energy density is dominated by* $V(\phi)$. In this regime, we have $\dot{\phi}^2/2 << V(\phi)$, $\rho_\gamma << V(\phi)$ and $|\ddot{\phi}| << |3H\dot{\phi}|$. Thus, by $p = \dot{\phi}^2/2 - V(\phi)$ and $\rho = \dot{\phi}^2/2 + V(\phi)$, we have

$$\dot{\rho} + p = 0. \tag{32}$$

For flat FRW case from Eqs. (7) and (31), we have

$$H = \sqrt{\frac{M}{3}\left(1 - \frac{\alpha}{\phi^n}\right)}. \tag{33}$$

From Eqs. (10) and (33), we have

$$\phi^{n/2+1}\sqrt{\phi^n - \alpha}\, d\phi = -\sqrt{\frac{M}{3}}\alpha n dt. \tag{34}$$

For $n=4$, we have

$$\phi(t)=\pm\left[\alpha+\left(8\sqrt{3M}\alpha\right)^{2/3}(t_2-t)^{2/3}\right]^{1/4} \tag{35}$$

where, t_2 is an integration constant.

For above value of $\phi(t)$, we have

$$a(t)=\exp\left(\frac{\phi^2(t)}{8}-\frac{\phi^6(t)}{24\alpha}\right). \tag{36}$$

The behaviour of scalar field ϕ with cosmic time t for the case 2.2.1, where the energy density is dominated by $V(\phi)$ can be seen in Fig.8, 9. Here, we consider some physically significant values of α, i.e. $\alpha=2.5$, 1.0, and 0.5. From Fig.8, it is observed that for all given choices of α, the scalar field ϕ is an increasing function of cosmic time t, if we consider the positive sign in the RHS of Eq. (35). The rapidity of growth of scalar field is observed at an early stage. Later on, this tends asymptotically to a constant for large time t. From Fig.9, it is observed that the scalar field ϕ is a decreasing function of cosmic time t,

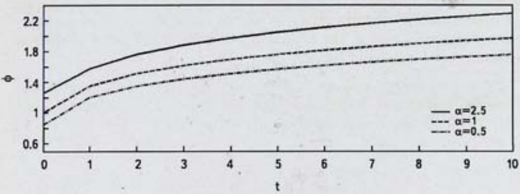

Fig.8. The scalar field ϕ with cosmic time t for $m=1$, $t_2=0$ in positive part of Eq. (35).

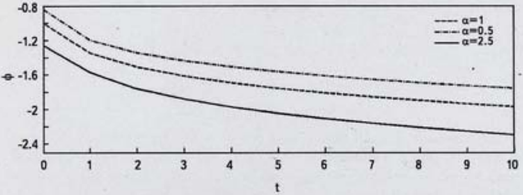

Fig.9. The The same as in Fig.8 for negative part of Eq. (35).

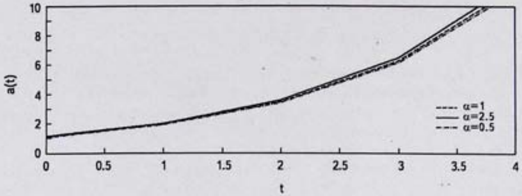

Fig.10. The scale factor $a(t)$ with cosmic time t for $m=1$, $t_2=0$ in Eq. (36).

if we consider the negative sign in the RHS of Eq. (35). The rapidity of decay of scalar field is observed at an early stage. Later on, this tends to asymptotically to a constant for large time t.

The behaviour of scalar factor a with cosmic time t for the case 2.2.1, where the energy density is dominated by $V(\phi)$ can be seen in Fig.10. From Fig.10, it is noticed that for all given choices of α, the scalar factor a is an increasing function of cosmic time t. The rapidity of growth of scalar factor is observed throughout the evolution of the universe.

2.2.2. *The energy density is dominated by* $\dot{\phi}^2/2$. Since energy density is dominated by kinetic energy density and there is no role of potential function $V(\phi)$. Therefore, this case is similar to the section 2.1.2.

2.2.3. *The regime when* $V(\phi)\approx\dot{\phi}^2/2$. In this regime, we neglect $3H\dot{\phi}$, then from Eq. (10), we have

$$\ddot{\phi}+\frac{dV(\phi)}{d\phi}=0. \tag{37}$$

From Eqs. (31) and (37), we have

$$\phi(t)=\left[\pm(n+2)\sqrt{\frac{M\alpha}{2}}\right]^{2/(n+2)}(t-t_3)^{2/(n+2)} \tag{38}$$

where t_3 is an integration constant and for $n=4$, we have

$$\phi(t)=\left[\pm 18M\alpha\right]^{1/6}(t-t_3)^{1/3}. \tag{39}$$

The behaviour of scalar field ϕ with cosmic time t for the case 2.2.3, where the energy density is dominated by $V(\phi)\approx\dot{\phi}^2/2$ can be seen in Fig.11. Here, we consider some physically significant values of α, i.e. $\alpha=2.5$, 1.0, and 0.5. From Fig.11, it is observed that for all given choices of α, the scalar field ϕ is an increasing function of cosmic time t. The rapidity of growth of scalar field is

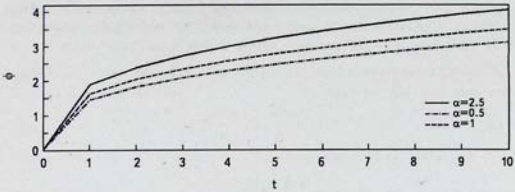

Fig.11. The scalar field ϕ with cosmic time t for $m=1$, $t_3=0$ in Eq. (39).

observed at an early stage. Later on, this tends asymptotically to a constant for large time t.

2.2.4. *Evolution of universe by energy density of matter/radiation* ρ_γ. In this regime, energy density of the universe is dominated by matter with equation of state $p_\gamma=\gamma\rho_\gamma$. This section is similar to the 2.1.4.

2.3. *Section C*. We consider

$$V(\phi)=V_0+\mu\left(\phi^2-M^2\right)^2. \tag{40}$$

2.3.1. *The energy density is dominated by* $V(\phi)$. In this regime, we have $\dot\phi^2/2<<V(\phi)$, $\rho_\gamma<<V(\phi)$ and $|\ddot\phi|<<|3H\dot\phi|$. Thus, by $p=\dot\phi^2/2-V(\phi)$ and $\rho=\dot\phi^2/2+V(\phi)$, we have

$$\rho+p=0. \tag{41}$$

For flat FRW case with conditions $|V_0|<<\left(\phi^2-m^2\right)^2$, from Eqs. (7) and (40), we have

$$H=\sqrt{\frac{\mu}{3}}\left(\phi^2-m^2\right) \tag{42}$$

$$3H\dot\phi+4\mu\left(\phi^2-m^2\right)\phi=0. \tag{43}$$

From Eqs. (42) and (43), we have

$$\phi(t)=\exp\left(4\sqrt{\frac{\mu}{3}}(t_4-t)\right) \tag{44}$$

where t_4 is constant of integration. From Eqs. (42) and (44), we have

$$a(t)=\exp\left(-\frac{1}{8}\exp\left(8\sqrt{\frac{\mu}{3}}(t_4-t)\right)-M^2\sqrt{\frac{\mu}{3}}t\right). \tag{45}$$

2.3.2. *The energy density is dominated by* $\dot{\phi}^2/2$. Since energy density is dominated by kinetic energy density and there is no role of potential function $V(\phi)$. Therefore, this case is similar to the section 2.1.2.

2.3.3. *The regime when* $V(\phi) \approx \dot{\phi}^2/2$. In this regime, we neglect $3H\dot{\phi}$, then from Eq. (10), we have

$$\ddot{\phi} + \frac{dV(\phi)}{d\phi} = 0. \tag{46}$$

From Eqs. (40) and (46), we have

$$\phi(t) = \frac{-2\sqrt{2}\, M \exp\left(2M\sqrt{\mu}\, t\right)}{\exp\left(4M\sqrt{\mu}\, t + 1\right)}. \tag{47}$$

2.3.4. *Evolution of universe by energy density of matter/radiation* ρ_γ. In this regime, energy density of the universe is dominated by matter with equation of state $p_\gamma = \gamma\rho_\gamma$. This section is similar to the 2.1.4.

2.4. *Section D.* We consider

$$V(\phi) = V_0 + a_1\phi^p \tag{48}$$

where, $a_1 = \nu M_{pl}^{4-p}$ with $p \geq 6$.

2.4.1. *The energy density is dominated by* $V(\phi)$. In this regime, we have $\dot{\phi}^2/2 << V(\phi)$, $\rho_\gamma << V(\phi)$ and $|\ddot{\phi}| << |3H\dot{\phi}|$. Thus, by $p = \dot{\phi}^2/2 - V(\phi)$ and $\rho = \dot{\phi}^2/2 + V(\phi)$, we have

$$\rho + p = 0. \tag{49}$$

For flat FRW case with condition $|V_0| << |a_1\phi^p|$, from Eqs. (7) and (48), we have

$$H = \sqrt{\frac{a_1}{3}}\phi^{p/2} \tag{50}$$

$$3H\dot{\phi} + 4\nu\left(\phi^2 - m^2\right)\phi = 0. \tag{51}$$

From Eqs. (50) and (51), we have

$$\phi(t) = \left(\sqrt{\frac{a_1}{3}}\, p\left(\frac{p}{2} - 2\right)\right)^{2/(4-p)} t^{2/(4-p)} \tag{52}$$

From Eqs. (51) and (52), we have

$$a(t) = \exp\left(b t^{4/(4-p)}\right) \tag{53}$$

where

$$b = \left(\frac{a_1}{3}\right)^{2/(4-p)} \left(\frac{4-p}{4}\right)\left(\frac{p^2}{2} - 2p\right)^{p/(4-p)}.$$

2.4.2. *The energy density is dominated by* $\dot{\phi}^2/2$. Since energy density is dominated by kinetic energy density and there is no role of potential function $V(\phi)$. Therefore, this case is similar to the section 2.1.2.

2.4.3. *The regime when* $V(\phi) \approx \dot{\phi}^2/2$. In this regime, we neglect $3H\dot{\phi}$, then from Eq. (10), we have

$$\ddot{\phi} + \frac{dV(\phi)}{d\phi} = 0. \tag{54}$$

From Eqs. (48) and (54), we have

$$\phi(t) = \left(\sqrt{-2a_1}\,\frac{2-p}{2}\right)^{2/(2-p)} (t - t_s)^{2/(2-p)} \tag{55}$$

where t_s is an integration constant. For $a_1 > 0$, no physically plausible solution is possible. However, for $a_1 < 0$, the Eq. (50) is contradicted. Threfore, no physically plausible solution is possible.

2.4.4. *Evolution of universe by energy density of matter/radiation* ρ_γ. In this regime, energy density of the universe is dominated by matter with equation of state $p_\gamma = \gamma\rho_\gamma$. This section is similar to the 2.1.4.

3. *Discussion and Conclusions*. The main goal of our work is to perform a general investigation of scalar field cosmology in theories with negative potentials. It is quite interesting that with an account taken of general relativity potentials that have minimum of $V(\phi) < 0$ can behave like potentials unbounded from below.

A general feature of all trajectories bringing the universe towards the singularity is that $\dot{\phi}^2/2$ becomes much greater than $V(\phi)$ near the singularity. This means that the description of the singularity is nearly model-independent, at least at the classical level. In particular, the equation of state of the universe approaching the singularity is $p = \rho$.

However the conclusion can be changed when an account is taken of quantum effects, including particle production near the singularity. The particle production near the singularity is so efficient that it turns off the regime $p = \rho$ when a contracting universe approaches the Planck density. The effects related to particle production are especially significant in an expanding universe as they tend to completely eliminate the stage with $p = \rho$.

In addition to the general study of cosmology with negative potentials, the investigation of the possibility that our universe can undergo repeated cycles of inflation and contraction (i.e. bounce and a cyclic universe) may be performed [10,11]. This scenario may allow us to combine attractive features of the oscillating universe model [12-16] and chaotic inflation [5].

The models allow for one simplification that resolves most of their remaining problems. If one removes the minimum of potential at $V(\phi)<0$, one returns to the usual scenario of chaotic inflation. It may describe an eternally self producing inflationary universe, as well as the present stage of accelerated expansion.

For most of our models, the scalar field $\phi(t)$ and the expansion parameter $a(t)$ are continuously increasing or decreasing functions of t and they tend to be constant, when t is very large.

Many other aspects of the present work is under our active consideration for a future study by dynamical system method and phase portrait. The cosmology for scalar fields with negative potentials and $\omega_\phi = p_\phi/\rho_\phi < -1$ leading to a collapsing universe is under investigation on the lines of the work by Macorra et al. [7]. We are considering to undertake a comprehensive study of cosmological models with negative potentials and there perturbation analysis in a future work. We will also study a fast-roll inflation in a universe [6].

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НЕКОТОРЫЕ КОСМОЛОГИЧЕСКИЕ МОДЕЛИ С ОТРИЦАТЕЛЬНЫМ ПОТЕНЦИАЛОМ

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Исследованы некоторые космологические модели, где эффективный потенциал $V(\phi)$ может стать отрицательным при некоторых значениях поля ϕ. Космологическая эволюция в моделях с минимумом при $V(\phi)<0$ аналогична в некоторых отношениях эволюции в моделях с неограниченными снизу потенциалами. В этом случае, вместо того, чтобы достичь режима AdS, в котором доминирует отрицательная энергия вакуума, вселенная может достичь точки поворота, где плотность энергии энергии исчезает, а затем она сжимается до сингулярности. В некоторых случаях такие модели могут привести к отскоку.

Ключевые слова: *Космологические модеи: отрицательный потенциал: отскок*

REFERENCES

1. *A.D.Linde*: Particle Physics and Inflationary Cosmology, Harwood Academic, Switzerland, 1990.
2. *D.H.Lyth, A.Riotto*: Phys. Rep., **314**, 1-146, 1999.
3. *A.R.Liddle, D.H.Lyth*, Cosmological Inflation and Large Scale Structure, Cambrige University Press, 2000.
4. *G.Fedler, A.Frolov, L.Kofman et al.*, LITA-2002-2004, hepth/0202017 Cosmology with Negative potentials, 2004.
5. *A.D.Linde*, Phys. Lett. B, **129**, 177, 1983.
6. *A.D.Linde*, JHEP. 0111, 052. [arxiv: hep-th/0110195] Fast-roll Inflation, 2001.
7. *A. de la Macorra, G.Germin*, [arxiv: astro-ph/0212148] Cosmology for scalar fields with negative potentials and $\omega_\phi < -1$, 2002.
8. *R.Kallosh, A.D.Linde, S.Prokushkin et al.*, arxiv: hepth/0110089, 2001.
9. *G.Felder, A.Frolov, L.Kofmani*, arxiv:hep-th/0112165. warped geometry of brane worlds, 2001.
10. *P.J.Steinhardt, N.Turok*, arxiv:hep-th/0111030. A cyclic model of the universe, 2001.
11. *J.Khoury, B.A.Ovrut, P.J.Steinhardt et al.*, arxiv:hep-th/0103239: The Ekpyrotic Universe: The colliding branes and the origin of hot big bang, 2001.
12. *R.C.Tolman*, Relativity, Thermodynamics and Cosmology. Oxford university Press., U.K, 1934.
13. *P.J.Steinhardt, N.Turok*, Endless Universe, Doubleday, New York, 2007.
14. *R.Penrose*, Cycles of Time, Bodley Head, London, 2010.
15. *M.Gasparini, G.Veneziano*, Phys. Rep., **373**, 1, 2003.
16. *P.H.Frampton*, Did Time Begin? Will Time End, World Sci. Publishers, Singapore, 2010.